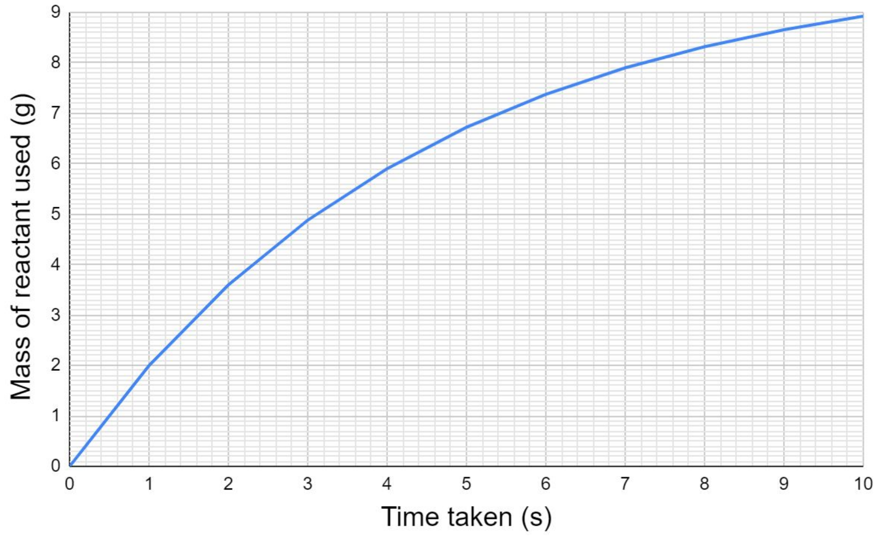
General mathematical skills

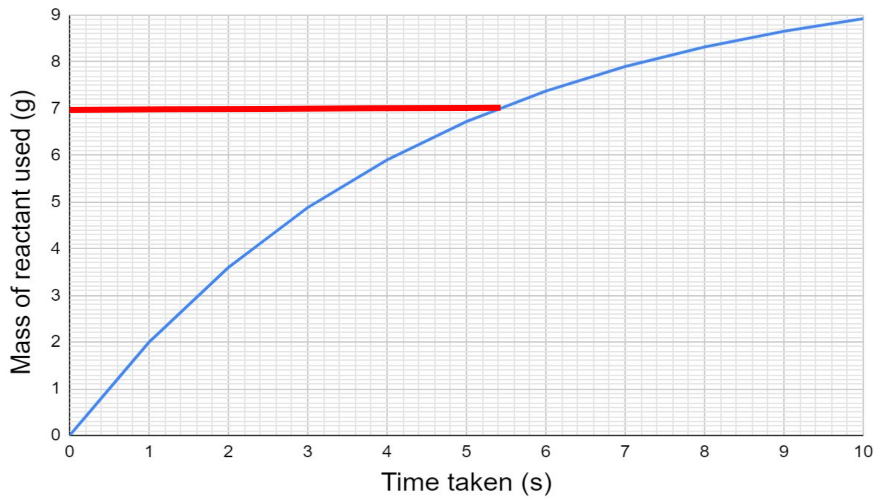
1. Interpreting graphs

1.1 Determining the value of one variable when given another and a graph linking the two

Example question - how long did it take for 7g of reactant to be used up in the experiment that has its results displayed in the graph to the right?

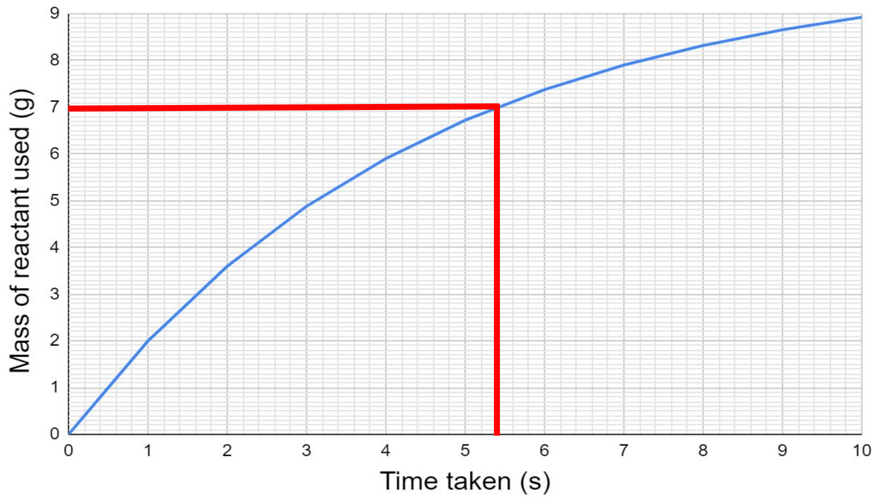


1. Draw a line (perpendicular (at 90°) to the axis) from the known value (7g) to the line on the graph



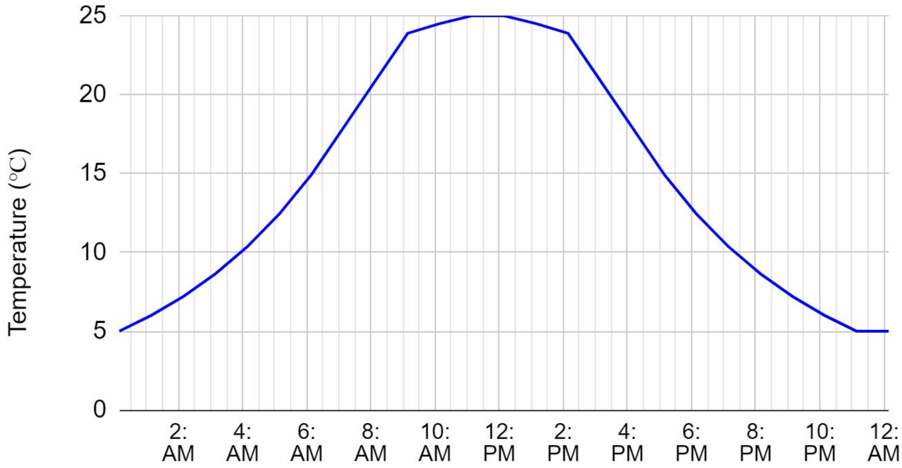
2. Draw a line perpendicular to the first to the other axis and read off the reading on that axis.

   In this example, it is 5.4s).

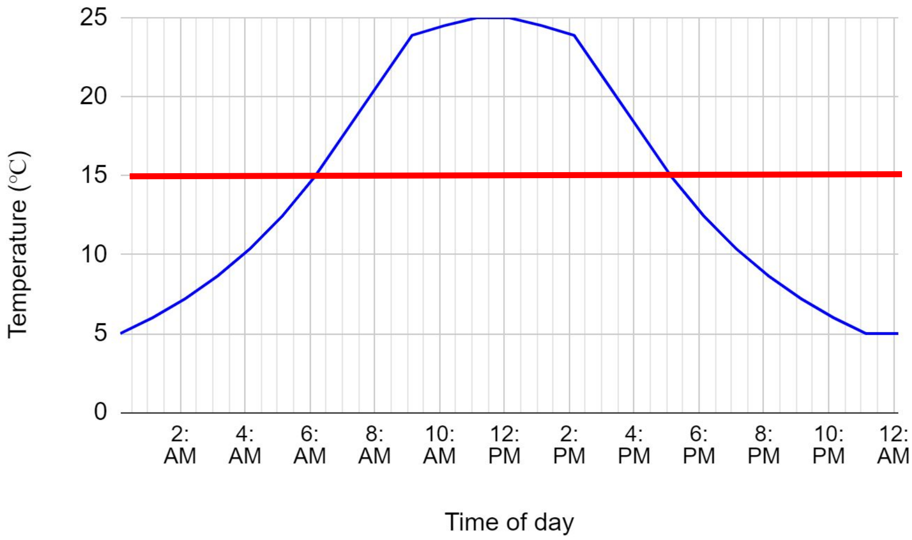


1.2 Using a graph to determine how long a variable was above (or below) a given value or between two given values - when given a graph of the variable against time

Example question - for how many hours was the temperature above 15 ℃ for the day that is displayed in the graph to the right?



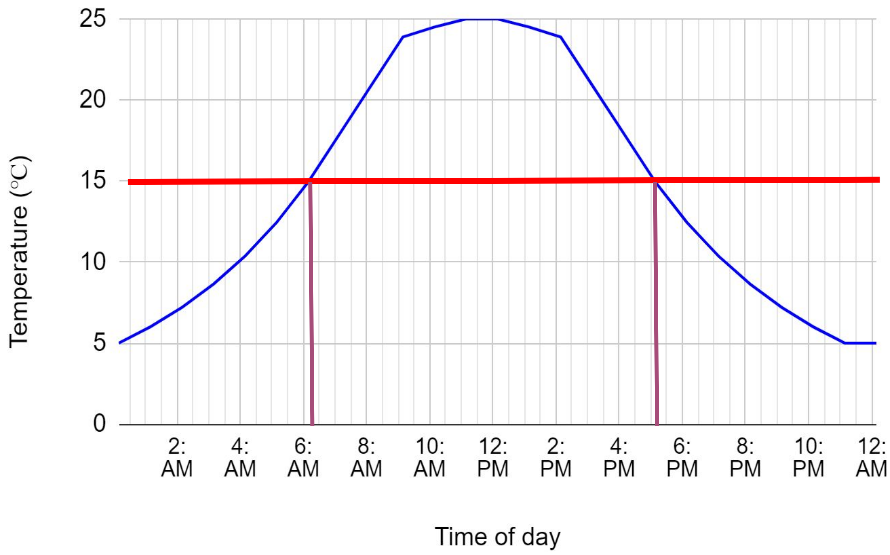
Draw a line perpendicular (at 90°) to the axis from the given value (15 ℃ in this example)



Draw lines perpendicular to the first lines at the points where it meets the line on the graph.

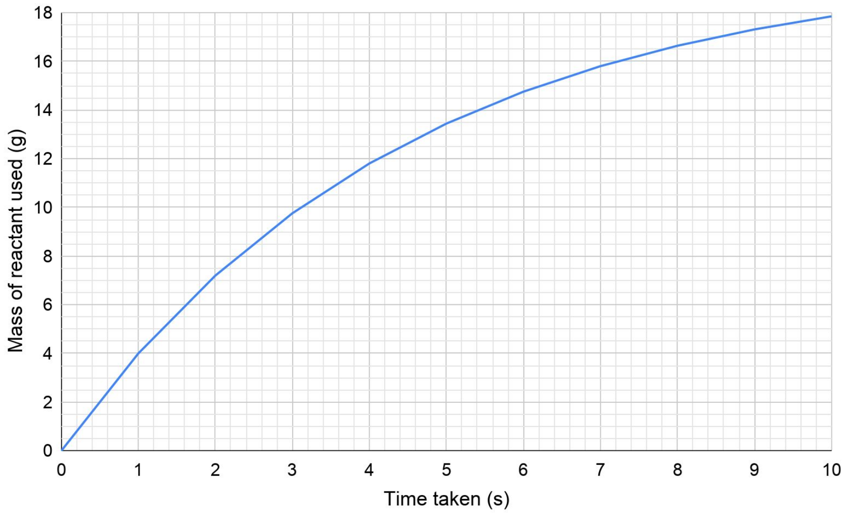
In this example, 6.15 am and 5.15pm and calculate the distance between them

= 11 hours



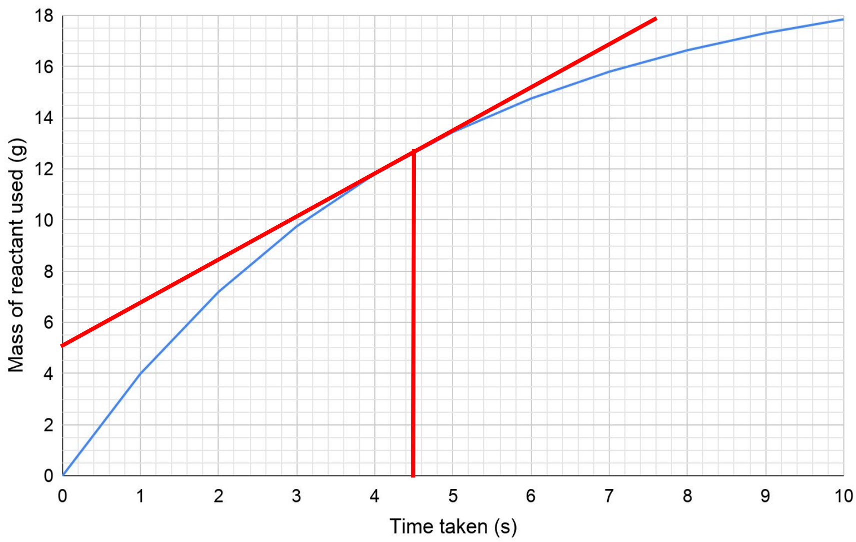
1.3 Calculate the rate at which a variable is changing is happening at a specific point in time when given a graph showing how the variable changes over time

Example question - what is the rate of reaction at 4.5s for the reaction displayed in the graph to the left?



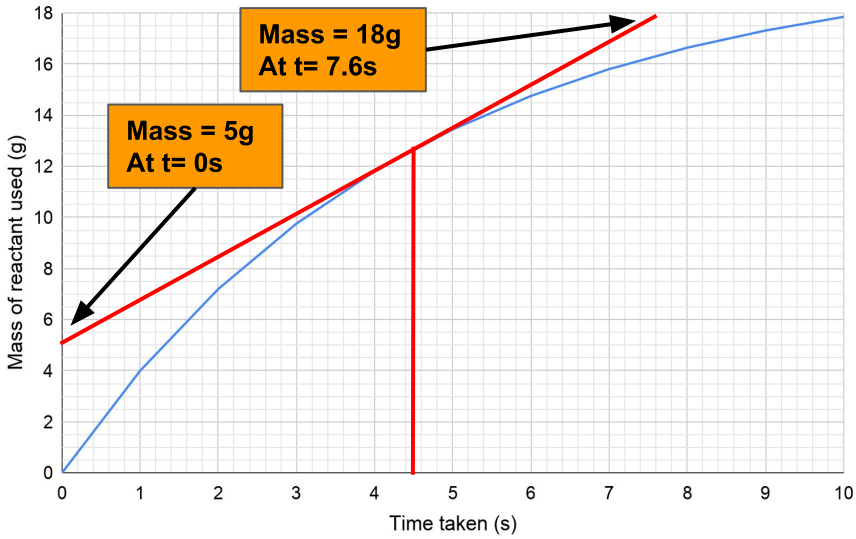
1. Draw a tangent to the line at the exact time mentioned in the question (4.5s).

   A tangent is a line that just touches the line.



2. Calculate the gradient of the tangent (by calculating the change in variable / change in time for 2 points on the line that are as far apart as possible.
   In the example shown we have point 1: 5g at 0s; point 2: 18g at 7.6s

   (18-5)/(7.6-0)
   = 13/7.6 = 1.71g/s



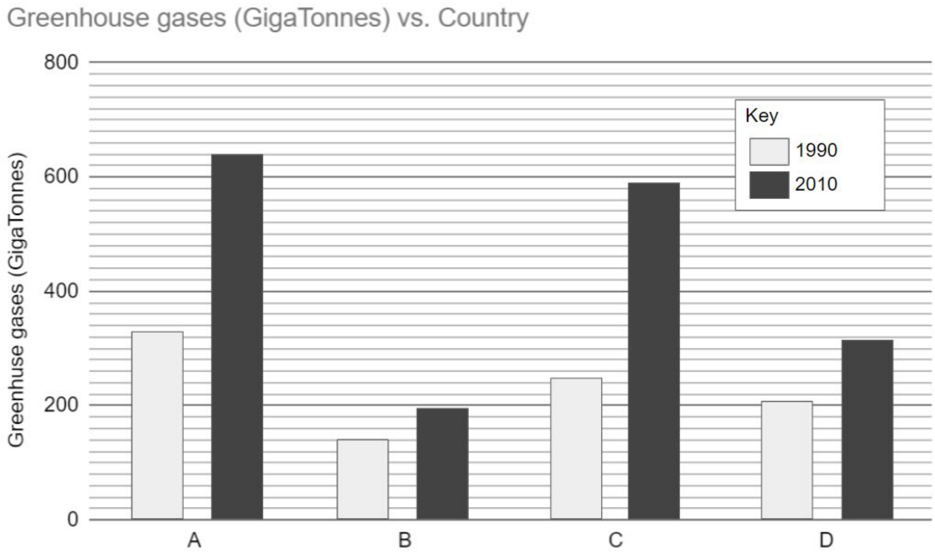
1.4 Comment on the truth (or otherwise) of a conclusion drawn from the evidence of the information on a graph - when asked to use the data from the chart to justify your answer (3-mark question).

Example question - The chart to the left shows the emission of greenhouse gases in four different countries in 1990 and 2010.

A student studied the chart and stated that "the countries emitted a lot more greenhouse gases in 2010 than they did in 1990."

Is the student correct?

Use the data from the chart to justify your answer (3 marks).



Model answer

1) Explain whether or not the data in the chart supports the point of view
   - In this example, the data in the chart supports the student's point of view.
2) Take information from the chart that supports your conclusion:
   - E.g. All of the countries increased the amount of greenhouse gases they emitted with A almost doubling the amount and C more than doubling..
3) Quote specific data from the chart
   - E.g. For example, C increased the amount of greenhouse gases from 250 GT in 1990 to 590 GT in 2010.

1.5 Explain why an anomalous / inaccurate result might have occurred

Example question - A student investigated the volume of carbon dioxide produced when different masses of calcium carbonate were reacted with 25cm^3 of dilute hydrochloric acid.

The student's results are shown in the table to the right..

The result for 0.32 g of calcium carbonate is anomalous.

Suggest what may have happened to cause this anomalous result.

Mass of calcium carbonate (g)	Volume of CO_2 gas (cm^3)
0.08	15
0.17	32
0.25	48
0.32	34
0.38	72
0.56	105
0.64	112
0.74	112
0.83	112

When asked a question on potential sources of error in a practical experiment, there are a few things to consider:

- The equipment used to measure might be inaccurate
    - E.g. digital balance may not be set to zero accurately.
- The equipment used to measure might not have a high enough resolution.
    - E.g. the scales might only measure to the nearest degree Celsius (when need to measure to the nearest 0.1 ℃).
- The equipment used to measure might have been used incorrectly.
    - E.g. eyes not level with when taking the reading on a thermometer.
- Other equipment might not have been used correctly.
    - E.g. bung not placed in a tube correctly - allowing gas to escape in equation above.
- There might be flaws in the method that was followed.
    - E.g. student might not have dried potatoes properly before weighing in the osmosis experiment.

When answering a question like this, vague answers like ‘he must have measured it wrong’ will not be accepted.

When given an anomalous value (like the one in the question above), it’s very important to suggest a potential problem with the method used THAT EXPLAINS THE DIRECTION OF ERROR (i.e. why it might be too high or too low).

In the example question displayed above, you would need to explain why the reading might be too low; e.g. the bung might not have been put in properly - allowing some of the gas to escape (or the student might have spilled some of the hydrochloric acid before adding it to the calcium).

1.6 Determining the resolution of a piece of equipment from the results

Example question - A student sets up an experiment to investigate the factors that affect the rusting of iron.

She set up six test tubes with slightly different characteristics and then measured the mass of each nail using a balance, left them all for a week and then measured the mass of each nail again with the same balance.

The table to the right shows the student's measurements.
What is the resolution of the balance the student used?

Test tube	Mass of nail (g)	
	At start	After a week
1	9.34	9.34
2	9.54	10.01
3	9.37	9.37
4	9.82	9.82
5	9.19	9.24
6	9.73	9.73

The resolution can be determined by looking at the lowest level of detail displayed in the results.

In this example, the results are displayed to the nearest 0.01g - so this is the resolution of the equipment.

If asked to present your answer in the form 1×10^ng, then look at the position of the lowest level of detail:

- If it is after the decimal point, then n = the number of places after the decimal point the lowest level appears:
    - In this example, the resolution = 1×10^{-2}g.
- If it is before the decimal point, then n = number of positions before the decimal point the lowest level of detail appears - 1.
    - E.g. if values were 60, 70 & 80g, then n = 2-1 = 1, so resolution = 1×10^1g (or 10g).

1.7 Calculating the percentage increase (or decrease) in a value

Percentage change = ((new value - initial value)/initial value) x 100%

Example question - A student sets up an experiment to investigate the factors that affect the rusting of iron.

She sets up six test tubes with slightly different characteristics (as displayed in the image to the left).

She then measured the mass of each nail using a balance, left them all for a week and then measured the mass of each nail again with the same balance.

The table directly above shows the student's measurements.

Test tube	Mass of nail (g)	
	At start	After a week
1	9.34	9.34
2	9.54	10.01
3	9.37	9.37
4	9.82	9.82
5	9.19	9.24
6	9.73	9.73

Calculate the percentage increase in mass after 1 week for the nails in test tube 2 and the nail in test tube 5.

Give your answer to three significant figures.

Percentage increase = ((new value - original value) / original value) x 100%

Nail 2; Percentage increase = ((10.01 - 9.54) / 9.54) x 100 = 4.9266 % or **4.93%**
Nail 5: Percentage increase = ((9.24-9.19) / 9.19) x 100 = **0.544%.**

1.8 Standard form

Standard form is a way of writing down very large or very small numbers easily.

When numbers are written in standard form:

- The first number is between 1 and 10.
- This is then multiplied by a factor of 10 to make the number

E.g. 5930 = 5.93×10^3 in standard form.
E.g. 2: 0.00572 = 5.72×10^{-3} in standard form

1.9 Calculate the percentage that one of the components makes up of the total

1) Calculate the total observations.
2) Percentage = (number of observations that fit profile / total observations) x 100%.

Example question - a student flicks a coin a number of times. The student gets 9 heads and 11 tails. What percentage of the students' coin tosses showed a head?

1) Total coin tosses = 9 + 11 = 20.
2) Total that fit profile (i.e. heads) = 9.
3) Percentage = (9 / 20) X 100%
   = <u>**45%**</u>

1.10 Designing an investigation; independent variables, dependent variables and control variables

Scientists often conduct investigations (experiments) to help them discover the relationship between two different variables; i.e. what impact does changing variable V_1 have on the second variable (V_2)?)

When designing the investigation, you must consider:

- The **independent variable** - what is the variable that is actively changed (V_1). This might be a variable that we choose to change:
  - a number of times to see a trend (e.g. the impact that adding weights has on the extension of a spring).
  - OR we may have two (or more groups) if the data is in categories (e.g. men / women or smokers / non-smokers).
- The **dependent variable** is V_2; the variable we measure to see what impact changing V_1 has had on it.
  - Can we measure it directly or do we measure other things that will allow us to calculate it?
- The **control variables** are the factors that must remain the same to ensure that any changes to V_2 only occur as a result to the changes in V_1 - rather than an as a result of changes to these factors.
- The action that must be followed to find the dependent variable for each value of the independent variable.
- What calculations must be done with the data that is measured determine the dependent variable?
- Accuracy / reliability / validity
  - What steps must be taken to make sure that the investigation's findings are repeatable?
    - Repeat, remove anomalies & calculate a mean OR
    - Make sure sample size is large enough for findings to be significant.

Example: a student investigated the impact that changing the amount of dilute hydrochloric acid that is added to 25cm^3 of sodium hydroxide solution has on the temperature change that results from the reaction.

- The independent variable (deliberately changed each time) is the amount of hydrochloric acid added each time.
- The dependent variable & what do we need to measure to be able to calculate it.
  - Temperature change - therefore need to measure the temperature before the acid is added and the temperature afterwards. `
- The control variables are:
  - The starting temperature.
  - The volume of sodium hydroxide solution.
  - The concentration of both substances.
- The action is adding the hydrochloric acid to the sodium hydroxide solution.
- What do we need to do to the measurements we have taken to calculate the dependent variable?
  - Subtract initial temperature from final temperature.
- Repeatability. Etc.
  - Repeat, remove anomalies & calculate a mean

1.11 Explaining the relationship between two variables from the data displayed on a graph (2 marks).

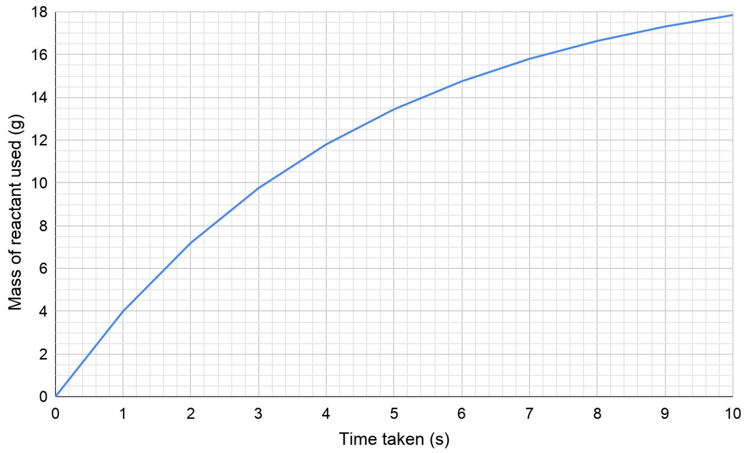
When asked to explain the relationship between two variables from a graph on a 2-mark question, make sure you explain:

1. Whether the second variable increases (or decreases) as the first variable increases.
2. Whether the rate of change increases or decreases as the first variable increases.

Example question - describe the trend displayed on the graph to the right (2 marks).

Answer:

- The longer the time taken, the more reactant is used up (1 mark).
- The rate at which the reactant is used up reduces over time. (2nd mark).



1.12 Compound measures - using the units to determine how to use values given in a question

The units on both sides of a calculation will always end up being the same on both sides of the equation.

If you are unsure of what to do with the information that is given on a question, then make sure that the calculations you use ends up with the same units on both sides.

Example question - a kitchen designed is trying to persuade a client that it's safe to have a marble worktop in their kitchen.

If marble has an activity of 770 Bq/kg and the proposed worktop has a mass of 80 kg, what would the activity of the kitchen worktop be (in Bq)?

Solution

The required units for the answer are Bq.

We have been given the activity of marble (Bq/kg) and the mass of the marble (kg).

If we multiply them together, then the units are:

$\frac{Bq}{\cancel{kg}} \times \cancel{kg} = Bq$ (as requested in answer - so this is what we must do)

Activity of worktop = 770 Bq/kg x 80 kg = 61,600 Bq

Solving an equation - when given the equation & all but one of the values

This page relates to how to find one variable (the target variable: ?) when given an equation and the other variables in the equation.

Follow the steps on this page

Enter the information from the question into the equation that links the variables

Example Q
How much energy does a 50W heater give out in 30 seconds?
Use the equation: Energy (E) = Power (P) x time (t)

Step 1 of answer - enter info provided into equation
E = 50 x 30

Is the variable that we are trying to calculate (?) on the top line (i.e. not dividing another number)?

No
E.g. 4 = 7 / ?

Multiply both sides of the equation by the variable to get it onto the top line

4 = 7 / ?
x ? x ?
4? = 7

Yes
E.g. 4? = 7

Is the variable that we are trying to calculate (?) on its own?

No
E.g. 3/? = 12

Get the target variable on its own by:

- Multiplying both sides of the equation by any number that the target variable (?) is currently being divided by.
- Dividing both sides of the equation by any number the target variable (?) is currently being multiplied by.

3?/4 = 12
x4 = x4
3? = 12 x 4
3? = 48
÷3 = ÷3
? = 48/3

Complete the calculation to determine the answer

? = 48/3
? = 16

Calculating a value to a number of significant figures

1) Start at the first non-zero number - that is the first significant figure.
2) Count back to the number of significant figures you are looking to calculate the number to.
    a) If the digit behind that digit is a 5 or more, round the last significant number up
    b) If not, leave it as it is.
3) Change any number that appears after the last significant number to 0.
4) That's it.

Example 2

Round **2**.3**6**8 to 3 significant figures.

1) The first significant figure is the **2.**
2) The last significant figure is **6**.
3) The digit after the **6** is an 8; this is higher than a 4 so round the **6** up to a 7.
4) Change everything after the last significant figure to zero.
5) Answer is 2.37

Example 2

Round **45**3,218 to 2 significant figures.

1) The first significant figure is the **4.**
2) The last significant figure is **5**.
3) The digit after the **5** is a 3; this is lower than a 5 so no need to round the 5 up.
4) Change everything after the last significant figure to zero.
5) Answer is 450,000

Example 3

Round 0.00**3**627 to 1 significant figure.

6) The first significant figure is the **3.**
7) The last significant figure is also the **3**.
8) The digit after the **3** is a 6; this is greater than a 5 so round the **3** up to a 4.
9) Change everything after the last significant figure to zero.
10) Answer is 0.004.

<u>Using the data in a table to explain the differences between two choices</u>

1) Make sure to use any numerical data that appears in the table to make numerical comparisons:
   a) E.g. A delivers twice as many [......] as B.
   b) E.g.2 Plastic bags take three times as long to decompose as . . .
2) Check every row/column in the table and make as many separate points as you can that could be relevant to the question that has been asked.

Example question

A student wants to buy a new mobile power source to charge her mobile phone.

She consults the table to the right & decide to buy the large phone.

Power source	Number of charges	Mass (g)
Small	2	50
Large	10	100
Extra large	20	300

Suggest why:

- It delivers 5 times as many charge as the small power source but is only half as heavy.
- It offers half the amount of charges as the extra large - but only weighs a third of the mass.

Calculating the mean, range and uncertainty in a set of numbers

1) Calculating the mean of a set of values
   a) Add all of the values together.
   b) Divide by the number of values provided.
2) Calculating the range of a set of values
   a) Range = difference between the highest number in a sample and the lowest number in a sample.
3) Uncertainty = ± (range/2).

Sample question - what is the mean, range and uncertainty of the following set of numbers?

(24, 26, 25, 27, 28)

Method	Worked example (data-set 24, 26, 25, 27, 28)
1) Mean = sum of readings / number of readings	= (24 + 25 + 25 + 27 + 28) / 5 = 26
2) Range = highest number - lowest number	= 28 - 24 = 4
3) Uncertainty = ± range/2	= ± (4/2) = ± 2
4) Answer = mean ± uncertainty	= 26 ± 2

Types of error

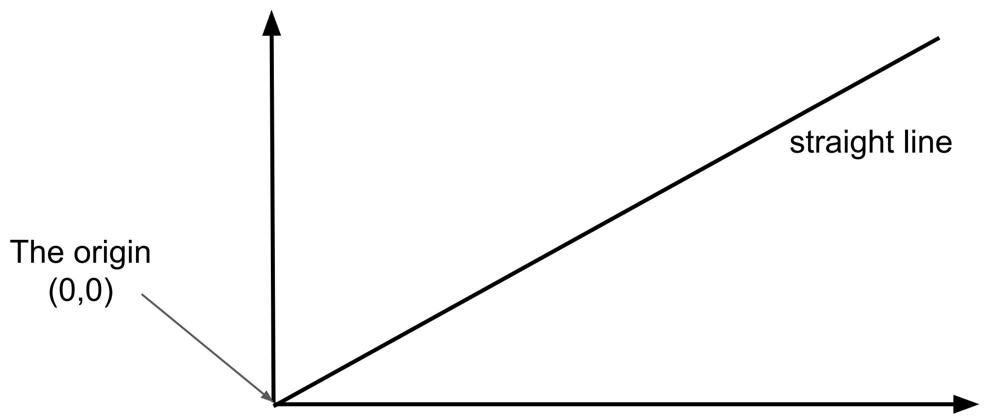
There are two main types of error:

- **Systematic** errors produce **consistent (predictable) errors in the same direction** (over or under by a fixed amount or proportion).
  - E.g. if the zero on a set of scales is set incorrectly.
  - These can be corrected by adjusting for the amount (or proportion) that they are out.
- **Random** errors:
  - Produce results that **out by different amounts (in both directions)** with no general pattern.
    - E.g. if the resolution on a piece of equipment is not detailed enough, your reading will be higher than the actual value as often as it is under.
  - These can be eliminated by:
    - Performing experiments multiple times, then removing anomalous results and calculating a mean (as the random errors should balance out if enough values are taken).

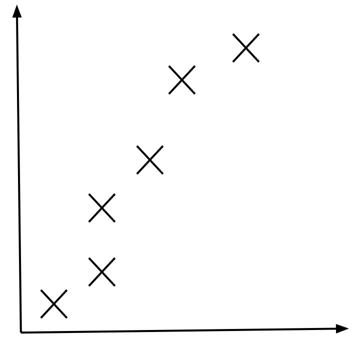
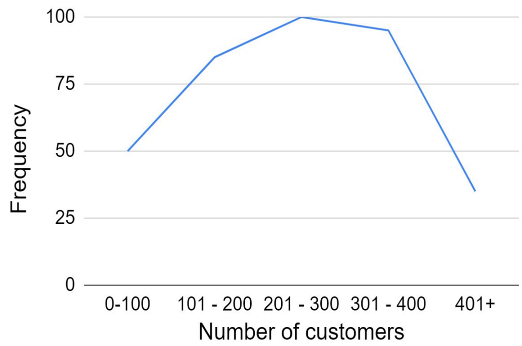
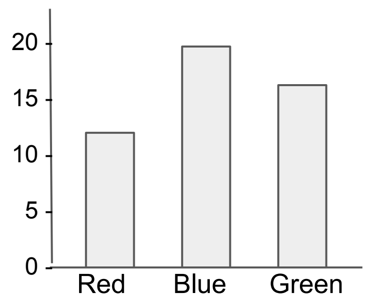
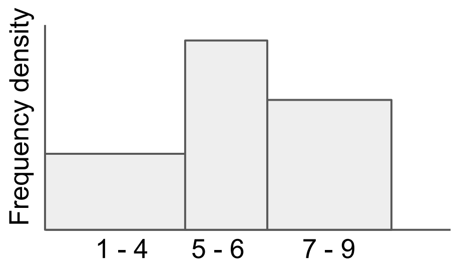
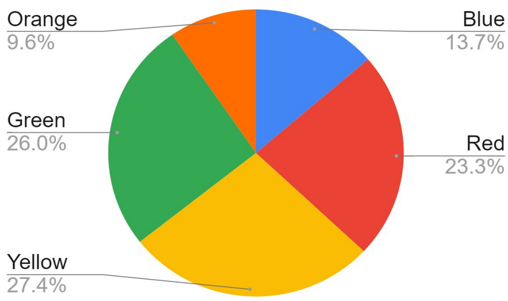
Directly proportional

If two variables are **directly proportional** to each other, then a graph of their relationship will be:

- A straight line . . .
- . . . that goes through the origin (the point 0,0).

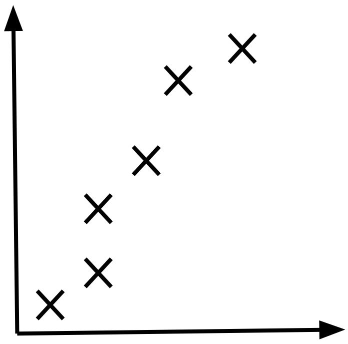


Types of chart

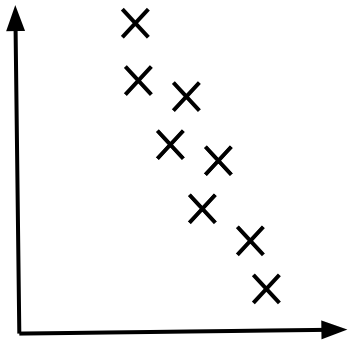
Type of chart	Purpose	Example
Scatter graph	Identifying potential correlations between two variables	
Frequency chart	Aggregating data on a single variable to make it easier to understand the relative distribution of values	
Bar chart	Similar to frequency chart but uses height of bars to show frequency - used to compare discrete (categoric) data; i.e. not continuous. There are gaps between the bars.	
Histogram	Like a bar chart but used to **compare continuous data**. **No gap** between bars which **can have different widths**. Y-axis is frequency density - so the area of the bar represents the frequency (not the height).	
Pie chart	Shows the relative proportion of each group as a fraction of the total	

Scatter graph

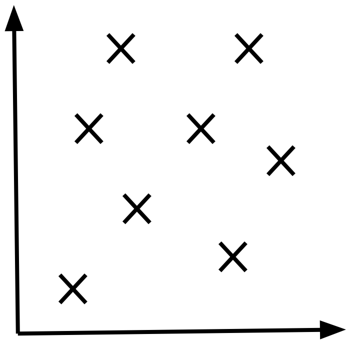
Scatter graphs are a good way of identifying a correlation between two variables.



Positive correlation - one variable tends to **in**crease when the other **in**creases



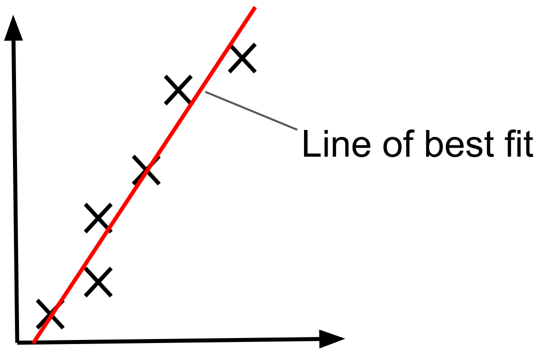
Negative correlation - one variable tends to **IN**crease when the other **DE**creases



No correlation - no obvious connection between the two variables

Line of best fit

The line of best fit is a line drawn on a graph that runs as close as possible to as many as points on the graph as possible.



Frequency tables and diagrams

Frequency tables are an efficient means of presenting a lot data on a single variable; e.g. the number of customers that visited a supermarket every day in a year would give 363 readings (as they are shut on Christmas Day & Easter Sunday.

Allocating the results into groups makes it much easier to see the patterns.

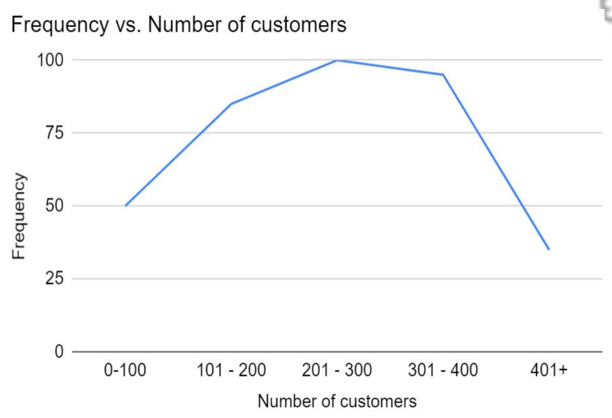
1) Decide on an acceptable set of groupings for the data so that the output will provide useful information:
    a) E.g. 1-100, 101-200, 201-300, 301-400,401+.
2) Allocate each of the readings to one of the groups
3) Count up the total number in each group.

Number of customers	Frequency
0-100	50
101 - 200	85
201 - 300	100
301 - 400	95
401+	35

Frequency diagram

A frequency diagram is a means of showing the results contained in a frequency table in graphical form.

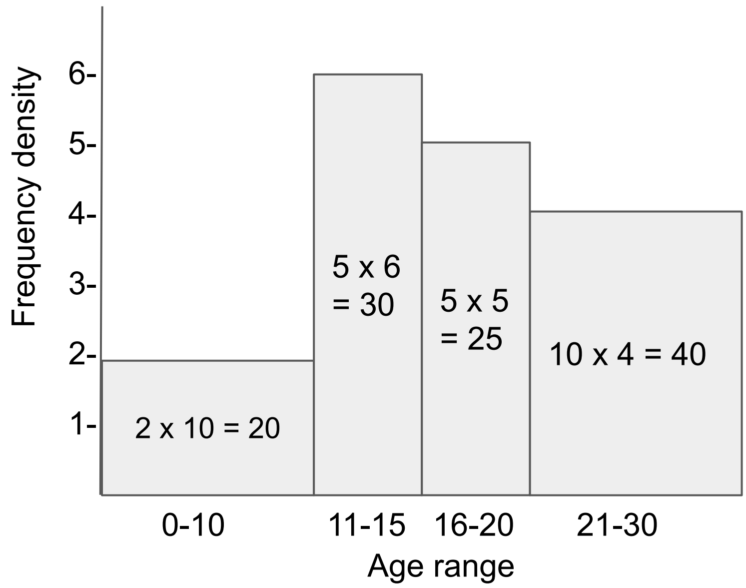
1) The groupings are displayed on the x-axis.
2) The frequency of each is displayed on the y-axis.



Histogram

Histograms are used to compare continuous data where the sizes of the segments may be different; e.g.

Age range	Frequency
0-10	20
11-15	30
16-20	25
21-30	40



The frequency for each bar is equal to the area of the bar.

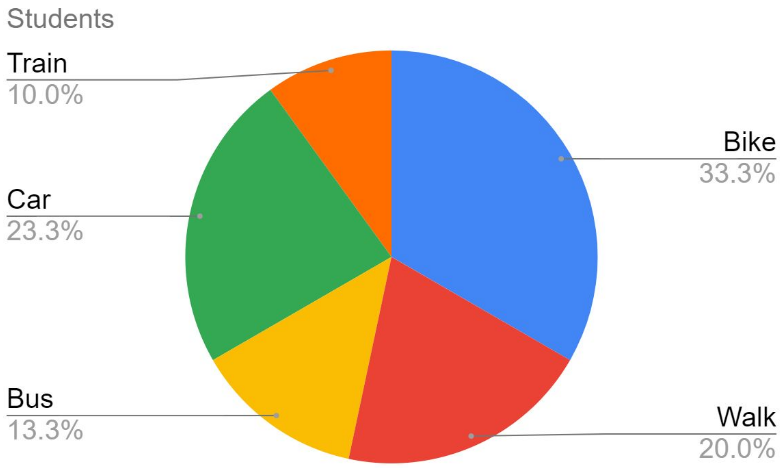
This is equal to the width of the bar x the frequency density (the height); e.g:

- Bar 1: width = 10; height = 2; frequency = 2 x 10 = 20.
- Bar 2: width = 5; height = 6; frequency = 6 x 5 = 30.
- Bar 3: width = 5; height = 5; frequency = 5 x 5 = 25.
- Bar 4: width = 10; height = 4; frequency = 4 x 10 = 40.

Pie Charts

Pie charts are used to show the relative proportions of different parts of a whole; e.g. the different means of transport for a class.

Means of transport	Students
Bike	10
Walk	6
Bus	4
Car	7
Train	3



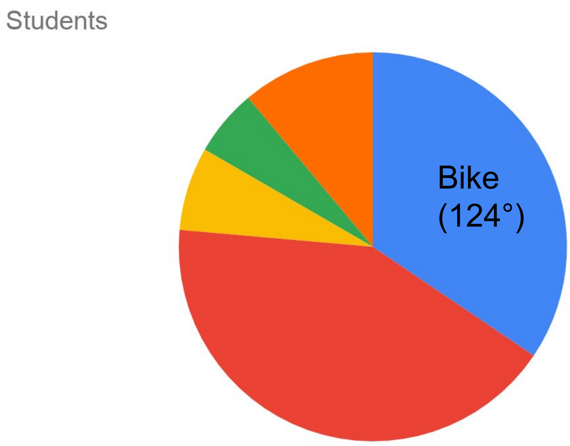
The size of the angle on each segment of the pie is equal to . . .
(The value of the variable/total of all values) x 360°

This is because there are 360° in a circle.

E.g. the angle on the train segment = (3/30) x 360 = 36°.

If you are given the angle of the segment and asked to calculate the frequency, then frequency = (angle/360) x total frequency.

Example question - there are 180 students in a school and their means of transport to school are shown in the bar chart below. How many students travel to school by bike?



Solution

The number travelling by bike =
(angle of segment/360) x total in sample
= (124/360) x 180
= 62

Repeating an experiment

Scientists repeat experiments to improve the accuracy.

They compare each set of results to check that they are all similar to each other.

This allows them to:

- Remove any anomalies (outliers). These are results that are too far away from the other results.
- Calculate a mean.

Comparing data in a table (or a graph)

When asked to draw conclusions (or compare or highlight similarities or differences between two variables), there are a number of things to consider:

1. Do the variables both change in the same direction (or do they move in different directions)?
2. Does one variable increase (or decrease) more than the other?
3. How do their rates of change compare?
    a. Is one increasing (or decreasing) faster than the other?
    b. Is this true throughout? Or does it change over time?

Design an investigation to test a hypothesis (or investigate a potential link between one factor and an outcome)

Example question - design an investigation to investigate the following hypothesis:

“Obese people are more likely to have type 2 diabetes than people who are not overweight.”

1) Have two groups of people - one with the attribute being investigated; one without it.
   a) E.g. obese / not obese.
2) The groups must be of equal size and large enough to be significant.
3) Decide what factors must be controlled to make sure that the investigation is fair:
   a) Age / gender
4) Decide what action must be taken to compare the two groups
   a) E.g Test for type 2 diabetes..

Making sure units in an equation are consistent with each other

It's really important when performing a calculation that you make sure the units in the variables given and the answer are all consistent with each other.

E.g If you are told that the specific latent heat of fusion of a substance is 5 J/g℃ and asked to calculate the energy required to melt 0.5 Kg of the substance, you cannot immediately use the following equation to calculate the answer:

Energy = mass x specific latent heat of fusion.

This is because the units are not consistent with each other:

- Specific latent heat is given in J/**g**℃,
- whereas the mass is give in **Kg**.

In this scenario, you must change one of the variables so that the units become consistent. In this example, you can either:

- Transform 0.5 Kg into g by multiplying by 1000 = 0.5 x 1000 = 500g
  - Then E = m x l = 500 x 5 = 2.500 J
- Or transform J/g℃ into J/kg℃ by multiplying by 1,000 - 5 x 1,000 = 5,000 J/kg℃
  - Then E = m xl = 5,000 x 0.5 = 2,500 J

Example question - a student wished to discover the specific heat capacity of a metal block. 4.8 KJ of energy was used to heat the block (which had a mass of 0.2kg) by 12℃.

What is the specific heat capacity of the block in J/**g**℃?

Use the equation:

Energy required to heat a substance (ΔE) = mass (m) x specific heat capacity (c) x change in temperature (Δ**ϴ**).

In this equation:

- The units of mass are not consistent - mass is given in **Kg**, but specific heat capacity in the answer is requested in J/**g**℃.
  - In this scenario, you must change the units of the variable provided in the question to match those required in the answer.
    - 0.2 **Kg** x 1,000 = 200 **g**
- The units of energy are not consistent - energy is given in **KJ**, but specific heat capacity in the answer is requested in J/g℃.
  - In this scenario, you must change the units of the variable provided in the question to match those required in the answer.
    - 4.8 **KJ** x 1,000 = 4,800 **J**
- The units of temperature are consistent (℃ in both).

1. Enter data provided into equation: ΔE = m x c x Δ**ϴ**
   4,800 = 200 x c x 12
   4,800 = 2,400 x c
2. Divide both sides by 2,400 to get c on its own
   C = 4,800/2,400 = 2 J/g℃